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Institution: Beijing-Dublin International College
Module: University Physics 2 (BDIC2008J)
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Problem Set 1

Electric Charge and Electric Field

Problem 1. *Of the charge Q initially on a tiny sphere, a portion q is to be transferred to a second, nearby sphere. Both spheres can be treated as particles and are fixed with a certain separation. For what value of q/Q will the electrostatic force between the two spheres be maximized?*

Solution. After the transfer, the charges on the two spheres are $Q - q$ and q. The magnitude of the electrostatic force between two charges q_1 and q_2 separated by a distance r is given by the Coulomb's law.

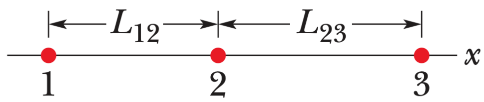
$$F = k\frac{q_1 q_2}{r^2}$$

where $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9 \mathrm{N \cdot m^2/C^2}$. In our case, $q_1 = Q - q$ and $q_2 = q$, so the magnitude of the force of either of the charges on the other is

$$F = \frac{1}{4\pi\varepsilon}\frac{q(Q - q)}{r^2}$$

We want the value of q that maximizes the function $f(q) = q(Q–q)$ by setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$. Thus, $q/Q = 0.5$. □

Problem 2. *Three charged particles lie on an x axis. Particles 1 and 2 are fixed in place. Particle 3 is free to move, but the net electrostatic force on it from particles 1 and 2 happens to be zero. If $L_{23} = L_{12}$, what is the ratio q_1/q_2?*



Solution. With rightward positive, the net force on q_3 is

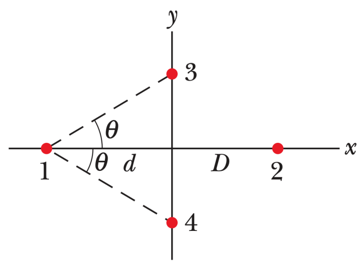
$$F_3 = F_{13} + F_{23} = k\frac{q_1 q_3}{(L_{12} + L_{23})^2} + k\frac{q_2 q_3}{L_{23}^2}$$

We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1, and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero $L_{23} = L_{12}$ and canceling k, q_3, and L_{12} leads to

$$\frac{q_1}{4} + q_2 = 0 \Rightarrow \frac{q_1}{q_2} = -4$$

□

Problem 3. *The figure shows an arrangement of four charged particles, with angle* $\theta = 30.0°$ *and distance* $d = 2.00\text{cm}$*. Particle 2 has charge* $q_2 = +8.00 \times 10^{-19}\text{C}$*; particles 3 and 4 have charges* $q_3 = q_4 = -1.60 \times 10^{-19}\text{C}$*.* ***(a)*** *What is distance* D *between the origin and particle 2 if the net electrostatic force on particle 1 due to the other particles is zero?* ***(b)*** *If particles 3 and 4 were moved closer to the x axis but maintained their symmetry about that axis, would the required value of D be greater than, less than, or the same as in part (a)?*



Solution. **(a)** We note that $\cos 30° = \sqrt{3}/2$, so that the dashed line distance in the figure is $r = 2d/\sqrt{3}$. The net force on q_1 due to the two charges q_3 and q_4 (with $|q_3| = |q_4| = 1.60 \times 10^{-19}\text{C}$) on the y axis has magnitude

$$2\frac{|q_1 q_3|}{4\pi\varepsilon_0 r^2}\cos 30° = \frac{3\sqrt{3}|q_1 q_3|}{16\pi\varepsilon_0 d^2}$$

This must be set equal to the magnitude of the force exerted on q_1 by $q_2 = 8.00 \times 10^{19}\text{C} = 5.00|q_3|$ in order that its net force be zero:

$$\frac{3\sqrt{3}|q_1 q_3|}{16\pi\varepsilon_0 d^2} = \frac{|q_1 q_2|}{4\pi\varepsilon_0 (D+d)^2} \Rightarrow D = d\left(2\sqrt{\frac{5}{3\sqrt{3}}} - 1\right) = 0.9245d$$

Given $d = 2.00\text{cm}$, this then leads to $D = 1.92\text{cm}$.

(b) As the angle decreases, its cosine increases, resulting in a larger contribution from the charges on the y axis. To offset this, the force exerted by q_2 must be made stronger, so that it must be brought closer to q_1 (keep in mind that Coulomb's law is inversely proportional to distance-squared). Thus, D must be decreased. □

Problem 4. *The magnitude of the electrostatic force between two identical ions that are separated by a distance of* $5.0 \times 10^{-10}\mathrm{m}$ *is* $3.7 \times 10^{-9}\mathrm{N}$*.* ***(a)*** *What is the charge of each ion?* ***(b)*** *How many electrons are "missing" from each ion (thus giving the ion its charge imbalance)?*

Solution. The magnitude of the electrostatic force between two charges q_1 and q_2, separated by a distance r is given by Coulomb's law. Let the charge of the ions be q. With $q_1 = q_2 = +q$, the magnitude of the force between the (positive) ions is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = k\frac{q^2}{r^2}$$

where $k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$.

(a) We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = (5.0 \times 10^{-10}\mathrm{m})\sqrt{\frac{3.7 \times 10^{-9}\mathrm{N}}{8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2}} = 3.2 \times 10^{-19}\mathrm{C}$$

(b) Let n be the number of electrons missing from each ion. Then, $ne = q$, or

$$n = \frac{q}{e} = \frac{3.2 \times 10^{-9}\mathrm{C}}{1.6 \times 10^{-9}\mathrm{C}} = 2$$

Electric charge is quantized. This means that any charge can be written as $q = ne$ where n is an integer (positive or negative), and $e = 1.6 \times 10^{-19}\mathrm{C}$ is the elementary charge. □

Problem 5. ***(a)*** *What equal positive charges would have to be placed on Earth and on the Moon to neutralize their gravitational attraction?* ***(b)*** *Why don't you need to know the lunar distance to solve this problem?* ***(c)*** *How many kilograms of hydrogen ions (that is, protons) would be needed to provide the positive charge calculated in (a)? (Hint: The mass of Earth is $M = 5.98 \times 10^{24}$kg, the mass of Moon is $m = 7.36 \times 10^{22}$kg, and the mass of ion is $m_i = 1.67 \times 10^{-27}$kg).*

Solution. **(a)** The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = G\frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q:

$$q = \sqrt{4\pi\varepsilon_0 GmM}$$

According to the Hint, $M = 5.98 \times 10^{24}$kg, and $m = 7.36 \times 10^{22}$kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{(6.67 \times 10^{-11}\text{N} \cdot \text{m}^2/\text{kg}^2)(7.36 \times 10^{22}\text{kg})(5.98 \times 10^{24}\text{kg})}{8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2}} = 5.7 \times 10^{13}\text{C}$$

(b) The distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

(c) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$C, so there must be

$$n = \frac{q}{e} = \frac{5.7 \times 10^{13}\text{C}}{1.60 \times 10^{-19}\text{C}} = 3.6 \times 10^{32}\text{ions}$$

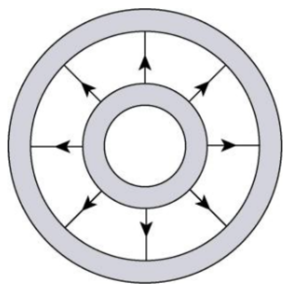
Since each ion has a mass of $m_i = 1.67 \times 10^{-27}$kg, so the total mass needed is

$$m = nm_i = (3.6 \times 10^{32})(1.67 \times 10^{-27}\text{kg}) = 6.0 \times 10^5\text{kg}$$

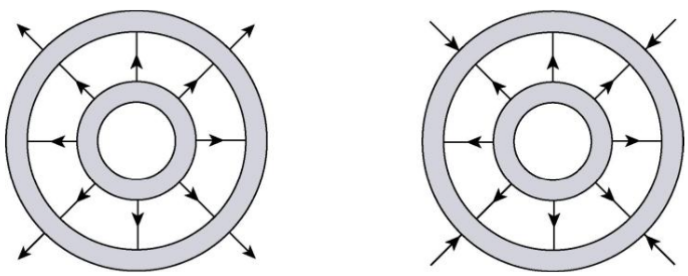
□

Problem 6. *Sketch qualitatively the electric field lines both between and outside two concentric conducting spherical shells when a uniform positive charge q_1 is on the inner shell and a uniform negative charge $-q_2$ is on the outer. Consider the cases $q_1 > q_2$, $q_1 = q_2$, and $q_1 < q_2$.*

Solution. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge that resides on the larger shell. The following sketch is for $q_1 = q_2$.

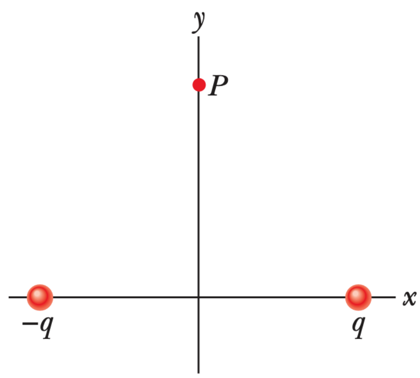


The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



□

Problem 7. *The figure shows two charged particles on an x axis: $-q = -3.20 \times 10^{-19}\text{C}$ at $x = -3.00\text{m}$ and $q = 3.20 \times 10^{-19}\text{C}$ at $x = +3.00\text{m}$. What are the **(a)** magnitude and **(b)** direction (relative to the positive direction of the x axis) of the net electric field produced at point P at $y = 4.00\text{m}$?*



Solution. **(a)** The vertical components of the individual fields (due to the two charges) cancel, by symmetry. Using $d = 3.00\text{m}$ and $y = 4.00\text{m}$, the horizontal components (both pointing to the $-x$ direction) add to give a magnitude of

$$\begin{aligned} E_{x,net} &= \frac{2|q|d}{4\pi\varepsilon_0(d^2+y^2)^{3/2}} = \frac{2(8.99 \times 10^9\text{N}\cdot\text{m}^2/\text{C}^2)(3.20 \times 10^{-19}\text{C})(3.00\text{m})}{[(3.00\text{m})^2 + (4.00\text{m})^2]^{3/2}} \\ &= 1.38 \times 10^{-10}\text{N/C} \end{aligned}$$

(b) The net electric field points in the $-x$ direction, or 180° counterclockwise from the $+x$ axis. □

Problem 8. *In Millikan's experiment, an oil drop of radius* $1.64\mu\mathrm{m}$ *and density* $0.851\mathrm{g/cm}^3$ *is suspended in the chamber when a downward electric field of* $1.92 \times 10^5\mathrm{N/C}$ *is applied. Find the charge on the drop, in terms of* e.

Solution. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = -qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$\begin{aligned} q &= -\frac{mg}{E} = -\frac{4\pi r^3 \rho g}{3E} = -\frac{4\pi(1.64 \times 10^{-6}\mathrm{m})^3(851\mathrm{kg/m}^3)(9.8\mathrm{m/s}^2)}{3(1.92 \times 10^5\mathrm{N/C})} \\ &= -8.0 \times 10^{-19}\mathrm{C} \end{aligned}$$

and $q/e = (-8.0 \times 10^{-19}\mathrm{C})/(1.60 \times 10^{-19}\mathrm{C}) = -5$, or $q = -5e$. $\square$

Problem 9. *A 10.0*g *block with a charge of* $+8.00 \times 10^{-5}$C *is placed in an electric field* $\vec{E} = (3000\vec{\imath} - 600\vec{\jmath})$N/C*. What are the* ***(a)*** *magnitude and* ***(b)*** *direction (relative to the positive direction of the* x *axis) of the electrostatic force on the block? If the block is released from rest at the origin at time* $t = 0$*, what are its* ***(c)*** x *and* ***(d)*** y *coordinates at* $t = 3.00$s*?*

Solution. **(a)** Using $\vec{F} = q\vec{E}$, we find

$$\begin{aligned}\vec{F} &= (8.00 \times 10^{-5}\text{C})(3.00 \times 10^{3}\text{N/C})\vec{\imath} + (8.00 \times 10^{-5}\text{C})(-600\text{N/C})\vec{\jmath} \\ &= (0.240\text{N})\vec{\imath} - (0.048\text{N})\vec{\jmath}\end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(0.240\text{N})^2 + (-0.048\text{N})^2} = 0.245\text{N}$$

(b) The angle the force F makes with the $+x$ axis is

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.048\text{N}}{0.240\text{N}}\right) = -11.3^\circ$$

measured counterclockwise from the $+x$ axis.

(c) With $m = 0.01$kg, the (x, y) coordinates at $t = 3.00$s can be found by combining Newton's second law with the kinematics equations. The x coordinate is

$$x = \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240\text{N})(3.00\text{s})^2}{2(0.01\text{kg})} = 108\text{m}$$

(d) Similarly, the y coordinate is

$$y = \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{(-0.048\text{N})(3.00\text{s})^2}{2(0.01\text{kg})} = -21.6\text{m}$$

□

Problem 10. *An electric dipole consisting of charges of magnitude* 1.50nC *separated by* 6.20μm *is in an electric field of strength* 1100N/C. *What are* ***(a)*** *the magnitude of the electric dipole moment and* ***(b)*** *the difference between the potential energies for dipole orientations parallel and antiparallel to* E*?*

Solution. The potential energy of the electric dipole placed in an electric field depends on its orientation relative to the electric field. The magnitude of the electric dipole moment is $\vec{p} = q\vec{d}$ where q is the magnitude of the charge, and d is the separation between the two charges. When placed in an electric field, the potential energy of the dipole is given by

$$U(\theta) = -\vec{p} \cdot \vec{E} = -pE\cos\theta$$

Therefore, if the initial angle between $\vec{p}$ and $\vec{E}$ is θ_0 and the final angle is θ, then the change in potential energy would be

$$\Delta U = U(\theta) - U_0(\theta) = -pE(\cos\theta - \cos\theta_0)$$

(a) With $q = 1.50 \times 10^{-9}$C and $d = 6.20 \times 10^{-6}$m, we find the magnitude of the dipole moment to be

$$p = qd = (1.50 \times 10^{-9}\mathrm{C})(6.20 \times 10^{-6}\mathrm{m}) = 9.30 \times 10^{-15}\mathrm{C} \cdot \mathrm{m}$$

(b) The initial and the final angles are $\theta_0 = 0$ (parallel) and $\theta_0 = 180°$ (anti-parallel), so we find ΔU to be

$$\Delta U = U(180°) - U(0) = 2pE = 2(9.30 \times 10^{-15}\mathrm{C} \cdot \mathrm{m})(1100\mathrm{N/C}) = 2.05 \times 10^{-11}\mathrm{J}$$

The potential energy is a maximum ($U_{max} = +pE$) when the dipole is oriented antiparallel to $\vec{E}$, and is a minimum ($U_{min} = -pE$) when it is parallel to $\vec{E}$. $\square$